NASA NsG-6-59

19p

N64-17583

Code 1

(NASA CR-53181; OTS:
NSF Grant G-14306

THE MOVING GRIFFITH CRACK OF UNVARYING SHAPE

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UNPUBLISHED PRELIMINARY DATA

OTS PRICE

XEROX $ 1.60 ph
MICROFILM $ 0.80 mf

ABSTRACT

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The propagating tensile fracture is treated as a steady state dynamic mixed boundary value problem in which the displacement at the fracture surface is specified. The displacement is made to approximate a Griffith crack at all velocities of fracture propagation. Although the shape of the stress distribution curves are identical to those determined by Yoffe, the magnitude of the tensile stress immediately ahead of the fracture front decreases with increase of velocity. The applied stress necessary to make a fracture of unvarying shape propagate at any particular velocity also decreases with increase of velocity. It appears that fractures of unvarying shape propagate at a single velocity, and start and stop without acceleration and deceleration. AUTHOR

1. INTRODUCTION

The propagating fracture problem has been treated by YOFFE (1951) and BILBY and BULLOUGH (1954) as a steady state dynamic boundary value problem. The fracture is initially in the form of a very flat ellipse (a Griffith crack) which moves across a medium under tension. The fracture is assumed to be of constant length, with the medium healing behind the fracture.

This model of a propagating fracture has two drawbacks. First, of course, a real medium never heals behind a fracture. Secondly, while the fracture maintains a constant length, its shape changes with velocity. The first of these drawbacks has been eliminated by YOFFE (1951) by studying the region in the immediate neighborhood of the fracture front of a very long fracture so that the events at the tail end have very little effect. The object of this paper is to examine the case of a moving Griffith crack the shape of which does not change with velocity. This is, in effect, a mixed boundary value problem in which the displacements are specified for the moving part of the system. An additional impetus to this study is that fact that displacements at real fracture surfaces appear to be small at all velocities.

2. SHAPE OF THE YOFFE CRACK

Let the fracture, of length 2a, move in the positive x direction with a velocity c (figure 1). The z axis is parallel to the fracture front, while the y axis is normal to the fracture surface. A stress $P_{yy} = T$ is applied at ∞. The stresses p_{ij} around the fracture are to be determined. From symmetry, we may study only the half-space y>o. If the applied stress T is removed then, for Yoffe's problem, the moving parts of the system have the following boundary conditions:

$$p_{ij} = 0 \quad \text{at} \quad \infty$$

and ~~for $-a \leq x' \leq a$~~ ~~where $x' = x - ct$~~

$$
\begin{aligned}
p_{yy} &= -P_{yy} = -T && \text{for } -a \leq x' \leq a,\ y = 0 \\
p_{xy} &= 0 && \text{for all } x',\ y = 0 \qquad \text{....(1)} \\
v &= 0 && \text{for } |x'| > a,\ y = 0
\end{aligned}
$$

where $x' = x - ct$, and
u and v are the displacements in the x and y directions respectively.

From equation (10) of Yoffe we find

$$v = \frac{T}{\mu}\left(\frac{2}{1+\beta^2} - 1\right)(a^2 - x'^2)^{1/2} \qquad \text{....(2)}$$

where c = fracture velocity

c_1 = dilatation wave velocity

c_2 = transverse wave velocity.

$$\gamma = (1 - c^2/c_1^2)^{1/2}$$

$$\beta = (1 - c^2/c_2^2)^{1/2}$$

$$\mu H = \frac{4\beta}{1+\beta^2} - \frac{(1+\beta^2)}{\gamma}$$

Expression (2) can be rewritten as

$$v = \frac{T}{\mu} \frac{(1-\beta^2)\gamma}{[4\beta\gamma - (1+\beta^2)^2]} (a^2 - x'^2)^{1/2} \quad \ldots.(3)$$

λ and μ are the Lame' constants.

Clearly, v depends on the fracture velocity c. The shape of the Yoffe crack at various velocities is shown in fig. 2. At c = o the shape is a flat ellipse and closely corresponds to a Griffith crack, but at higher velocities the Yoffe crack is no longer a flat ellipse and can no longer be approximated to a Griffith crack. At the branching velocity, for instance, $v\Big|_{x'=0} = 1{\cdot}14\, a \cdot \frac{T}{\mu}$

for a medium with a Poisson's ratio of 0.25. As Yoffe sets no constraint on the value of T/μ we assume the ratio is not very small.

3. STATEMENT OF THE PROBLEM

A tensile fracture is here defined as a fracture caused by purely tensile stresses acting at or near the fracture front and surface. Hence the boundary conditions for a tensile fracture of constant shape and length are, on y = o,

$$\begin{aligned} p_{xy} &= o \qquad \text{for all } x' \\ v &= o \qquad \text{for } |x'| > a \\ v &= n(a^2 - x'^2)^{1/2} \quad \text{for } |x'| < a \end{aligned} \qquad \ldots\ldots (4)$$

where n, the shape factor, is a small arbitrary constant. This last expression insures that the fracture always approximates a flat ellipse.

The solution can be obtained in a manner analogous to those of YOFFE (1951) and BILBY and BULLOUGH (1954). We assume displacements of the type

$$u = \int_0^\infty A(s)\left[\exp(-\gamma s y) - \frac{2\beta\gamma}{1+\beta^2}\exp(-\beta s y)\right]\sin sx'\,ds$$

$$\ldots\ldots (5)$$

$$v = \int_0^\infty \gamma A(s)\left[\exp(-\gamma s y) - \frac{2}{1+\beta^2}\exp(-\beta s y)\right]\cos sx'\,ds$$

where A(s) is an unknown function to be determined from the boundary conditions. As y = o the expression for v becomes

$$v = \frac{\gamma(\beta^2-1)}{(\beta^2+1)}\int_0^\infty A(s)\cos sx'\,ds$$

or $$\int_0^\infty A(s)\cos sx'\,ds = 0 \qquad \text{for } |x'| > a$$

$$\int_0^\infty A(s)\cos sx'\,ds = n\,\frac{(\beta^2+1)}{(\beta^2-1)\gamma}\,(a^2-x'^2)^{1/2} \qquad \text{for } |x'|<a \quad \ldots.(6)$$

A comparison of the dual-integral equations (6) with equations (7) of Yoffe shows that a solution of A(s) will be similar to that of Yoffe, if the factor $-\frac{T}{H}$ is replaced by $\frac{n\,(\beta^2+1)}{(\beta^2-1)}$.

The expressions for the stresses are then, in the notation of BILBY and BULLOUGH (1954)

$$p_{xx} = nL.\left\{\gamma m_1 \mathcal{R}\left[1-f(z_1)\right] + \frac{4\mu\beta}{(1+\beta^2)}\mathcal{R}\left[1-f(z_2)\right]\right\}$$

$$p_{yy} = nL.\left\{-\gamma m_2 \mathcal{R} f(z_1) + \frac{4\mu\beta}{(1+\beta^2)}\mathcal{R} f(z_2)\right\} \quad \ldots.(7)$$

$$p_{xy} = nL.\left\{-2\mu\,\mathcal{I} f(z_1) + 2\mu\,\mathcal{I} f(z_2)\right\}$$

where

$$L = \frac{(\beta^2+1)}{(\beta^2-1)}$$

$$m_1 = \mu\left[(\beta^2-1) - 2\gamma^2\right]/\gamma^2$$

$$m_2 = \mu\left[(\beta^2-1) + 2\right]/\gamma^2$$

$$z_1 = x' + i\gamma y$$

$$z_2 = x' + i\beta y$$

$$f(z) = z/(z^2-a^2)^{1/2}$$

The expressions for the stresses can be considerably simplified if one concentrates only on the region in the immediate neighborhood of the fracture front. Let a coordinate system (r, θ) be defined such that

$$x' - a = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

and let

$$z_0 = a + r\exp(i\theta) = a\left[1 + k_0\exp(i\theta)\right]$$

$$z_1 = a\left[1 + k_1\exp(i\theta_1)\right]$$

$$z_2 = a\left[1 + k_2\exp(i\theta_2)\right]$$

where

$$k_1 = k_0\left[1 - (c^2/c_1^2)\sin^2\theta\right]^{1/2}$$

$$k_2 = k_0\left[1 - (c^2/c_2^2)\sin^2\theta\right]^{1/2}$$

$$\tan\theta_1 = \gamma\tan\theta$$

$$\tan\theta_2 = \beta\tan\theta$$

For $(r/a) \ll 1$ we have

$$f(z_n) \simeq g(\theta_n, k_n) = \frac{\exp(-i\theta_n/2)}{\sqrt{2k_n}}$$

The expressions for the stresses in the present case are, thus,

$$p_{xx} = \frac{n\mu}{\sqrt{2k_0}} L\left[-\Upsilon m_1 \frac{\cos(\theta_1/2)}{\sqrt{k_1/k_0}} - \frac{4\beta}{(1+\beta^2)}\frac{\cos(\theta_2/2)}{\sqrt{k_2/k_0}}\right]$$

$$p_{yy} = \frac{n\mu}{\sqrt{2k_0}} L\left[-\Upsilon m_2 \frac{\cos(\theta_1/2)}{\sqrt{k_1/k_0}} + \frac{4\beta}{(1+\beta^2)}\frac{\cos(\theta_2/2)}{\sqrt{k_2/k_0}}\right] \quad \ldots\ldots (9)$$

$$p_{xy} = \frac{n\mu}{\sqrt{2k_0}} L\cdot 2\left[\frac{\sin(\theta_1/2)}{\sqrt{k_1/k_0}} - \frac{\sin(\theta_2/2)}{\sqrt{k_2/k_0}}\right]$$

The stresses $p_{\theta\theta}$, p_{rr} and $p_{r\theta}$ are then calculated from p_{xx}, p_{yy} and p_{xy} with the help of well known transformation relations (DURRELLI <u>et al.</u> 1958).

The magnitude of the stress $P_{YY} = T$ that has to be applied to make the fracture propagate at any particular velocity can be easily calculated and is given

$$P_{yy} = n\mu\left[\frac{4\beta\Upsilon - (1+\beta^2)^2}{(\beta^2-1)\Upsilon}\right] \quad \ldots\ldots (10)$$

4. RESULTS

The stresses $p_{\theta\theta}$, p_{rr} and $p_{r\theta}$ were computed on the Rice Computer. The values of the independent variables used are: Poisson's ratio $\sigma = 0.25$; relative fracture velocity c/c_2 from 0.20 to 0.80; and θ from 0° to 90°. The curves are presented in figure 3. The applied stress P_{yy} was also computed on the Rice Computer. Figure 4 shows the change in magnitude of P_{yy} with increasing velocity.

The shapes of the curves in figure 3 are identical to those of YOFFE (1951) and differ only in magnitude. The branching velocities if such velocities exist, are identical to those given in table 2 of MANSINHA (1964).

In the present case the stresses depend on μ, the shear modulus In Yoffe's problem the stresses were independent of all elastic constants with the exception of Poisson's ratio. It would appear at first sight that the results are independent of n, the shape factor, as n appears as a common factor in all the expressions p_{ij}. If true, this would mean that the results would be valid for a moving disturbance of any unvarying shape. Such is not the case because n has to be small for the boundary conditions to be valid. Consequently the propagating fracture will always approximate a flat ellipse.

5. DISCUSSION

From a study of figures 3 and 4 it will be presently shown that a fracture of constant shape will tend to propagate at a single velocity c, which is the same as the maximum velocity, c_m. It may safely be assumed that $p_{\theta\theta}$ must attain a minimum value τ at any point (r_0, θ) ahead of the fracture before the fracture can propagate. Thus the fracture can move forward only if $p_{\theta\theta} \geq \tau$. The fracture will come to a stop if $p_{\theta\theta} < \tau$.

From figure 3 it can be seem that $p_{\theta\theta}$ decreases in magnitude with increase of velocity. Therefore when $c > c_m$, $p_{\theta\theta} < \tau$ over all θ, and the fracture will stop suddenly. Clearly then c_m is the maximum velocity of propagation for a fracture of constant shape. This velocity c_m will, in general, be different from the branching velocity of YOFFE (1951).

The velocity c_m is also the <u>only velocity</u> at which a fracture of constant shape propagates. Referrring to figure 4, it is seen that the applied stress $P_{yy} = T$ required to maintain a fracture of constant shape moving at any velocity decreases with increase of velocity. The magnitude of $P_{yy} = T$ at $c = 0$ is interpreted as the stress required to form and maintain a Griffith crack of shape $n(a^2 - x'^2)^{1/2}$. For our purpose let us assume that such a static Griffith crack already exists in an unstressed elastic body. Let a gradually increasing stress $P_{yy} = T$ be applied to the body. From figure 4 it is clear that for low values of P_{yy} the fracture should move at high velocities. However, until P_{yy} reaches a value, say T_m, such that $p_{\theta\theta} = \tau$, the fracture will be static. When $p_{\theta\theta} = \tau$ the fracture will move forward at a velocity c_m.

Let us suppose that the stress $P_{yy} = T$ continue to increase even after reaching the value $T = T_m$. Since this magnitude of P_{yy}

is greater than that appropriate to a velocity c_m and the shape factor n, two effects are possible. From expression (10) it is clear that an increase in P_{yy} will mean an increase in n, or a decrease in c, or both. If n is constant, and c decreases, from figure 3 it will be seen that $P_{\theta\theta} \gg \mathcal{E}$ over a large range of angles. The fracture will lose energy by branching and will therefore revert back to a velocity c_m. If c is constant and n changes then the entire set of curves in figure 3 will be shifted upwards. If the change in n is sufficient, Yoffe's analysis will be more pertinent to the problem, and the fracture will lose energy by branching after reaching the branching velocity c_b.

Experimental evidence for the above set of conclusions is not lacking. Most fractures have not been observed to change shape significantly during propagation. In many cases fractures appear to be able to reach a maximum velocity without branching (SCHARDIN, 1959). SCHARDIN (1959) has also observed that no fracture moving at the maximum velocity has been observed to decelerate. Experimentally observed velocities which are lower than the maximum may be due to a succession of stops and starts or plasticity effects. The former has been experimentally observed (MANSINHA, 1962). Extensive plastic deformation occurs at the tip of static cracks because high stresses act at the tips for a relatively long time. Further, as no physical mass is moving at the fracture velocity, it is not impossible for a fracture to start and stop without acceleration or deceleration.

The main limitation of the present analysis is the fact that quantitative estimates for c_m and τ cannot be made from the theory, because of the presence of the arbitrary constants n and k_o in the expressions for p_{ij}. However, experimental test of the qualitative conclusions can be made with high resolution equipment to determine (1) if the stress field ahead of a fracture moving at the maximum velocity changes just before the fracture stops; (2) if the displacement at fracture surface changes with velocity and (3) if low velocity fractures are essentially different from those moving at the maximum velocity.

6. CONCLUSION

The propagating tensile fracture may be approximated by a moving Griffith crack whose shape does not change with velocity. The maximum velocity of this type of fracture is determined by the fact that the stress ahead of the fracture front decreases with velocity. The applied stress necessary to make the fracture propagate at any velocity decreases with increase of velocity. These characteristics imply that a moving Griffith crack of unvarying shape moves at a single constant velocity, without acceleration or deceleration.

ACKNOWLEDGEMENT

I wish to thank Dr. J. Cl. De Bremaecker for his interest in the work and many interesting discussions. This work was supported by Grants NsG-6-59 of the National Aeronautics and Space Administration and G-14306 of the National Science Foundation.

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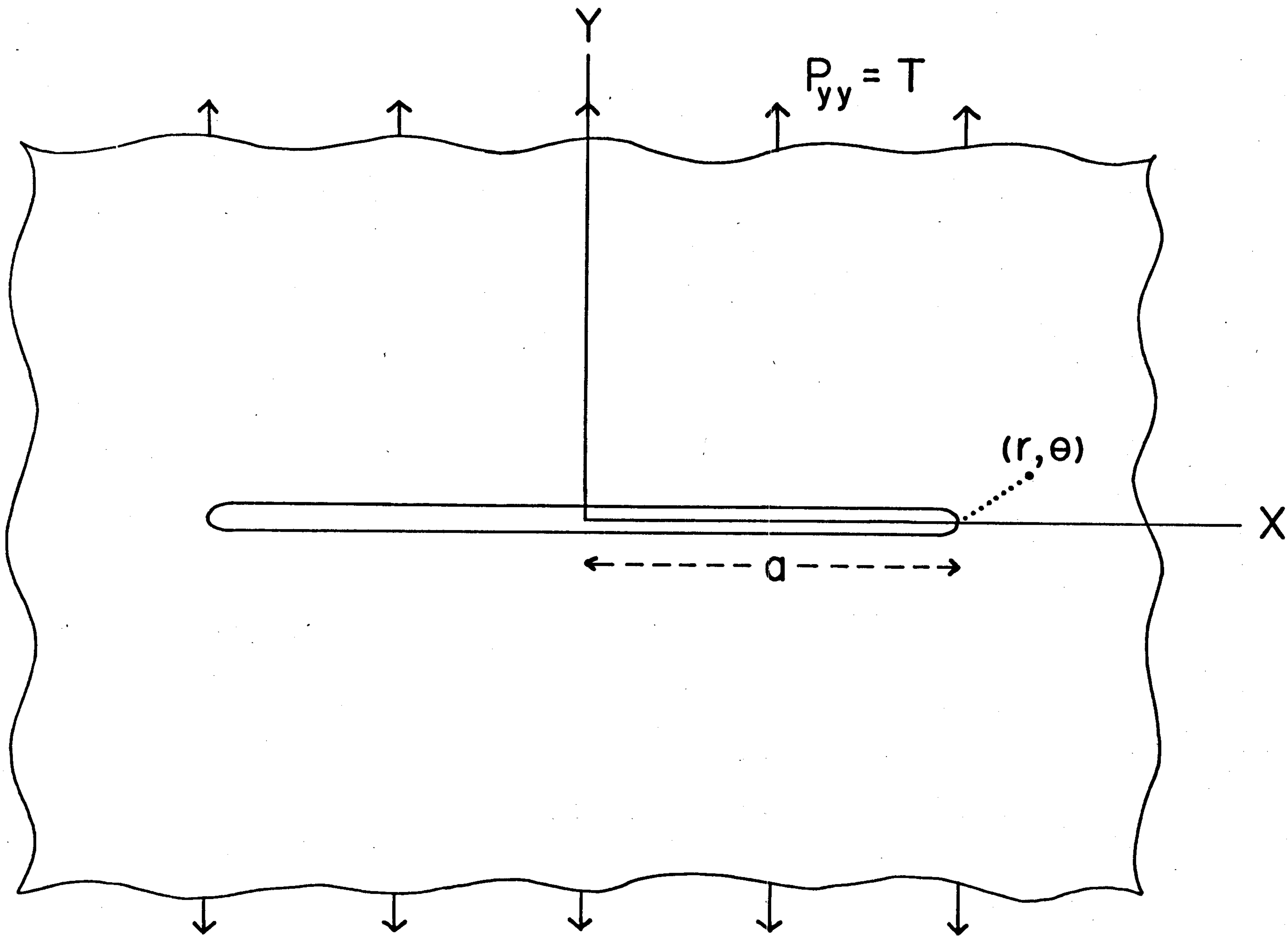

Fig. 1   Griffith crack of length 2a moves in the x direction. The y axis is normal to fracture surface. The radius r is much smaller than a.

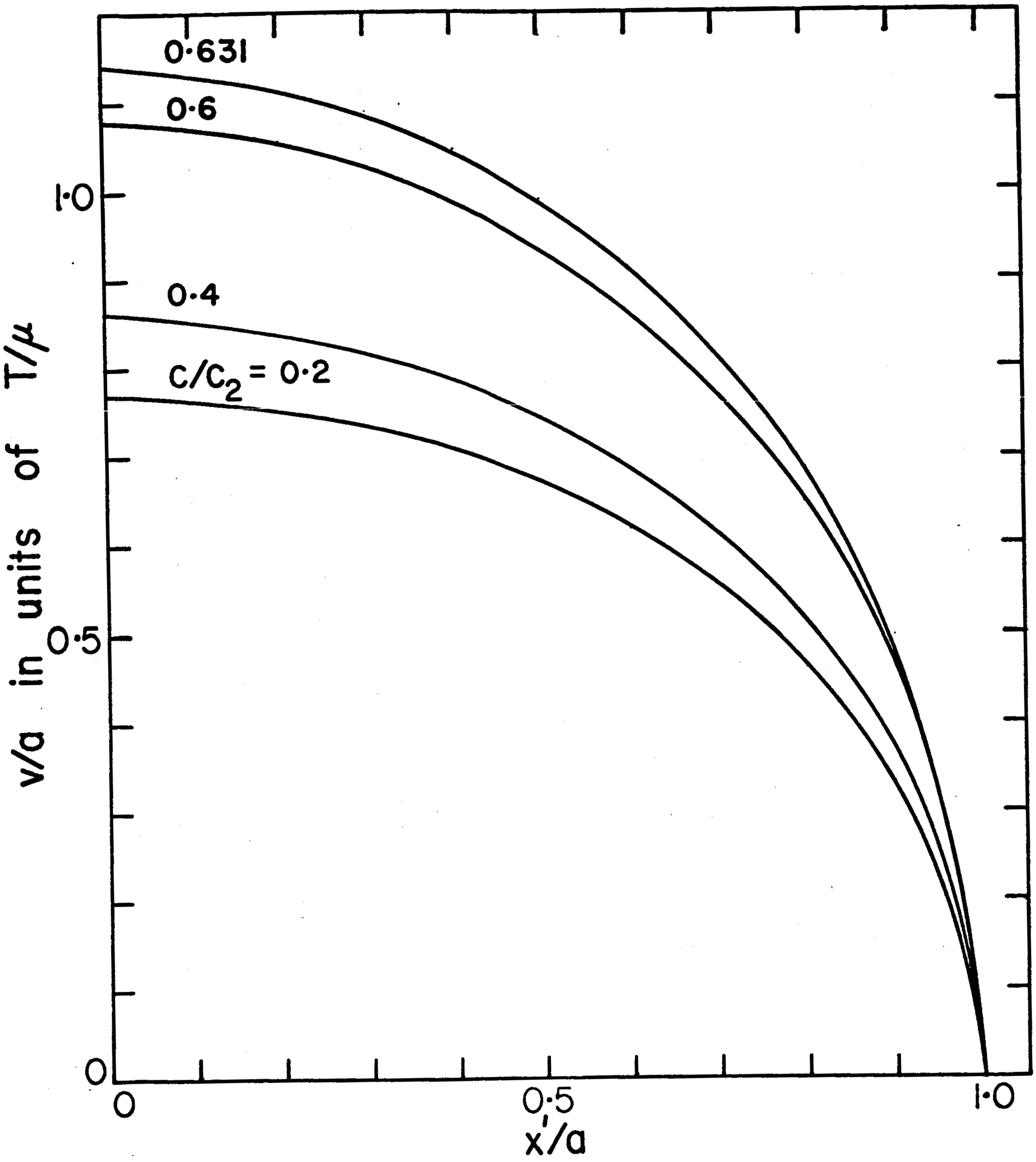

Fig. 2. Shape of the Yoffe crack at various velocities. At zero velocity the shape is flat and coincident with the x axis. Poisson's ratio of the medium is 0.25.

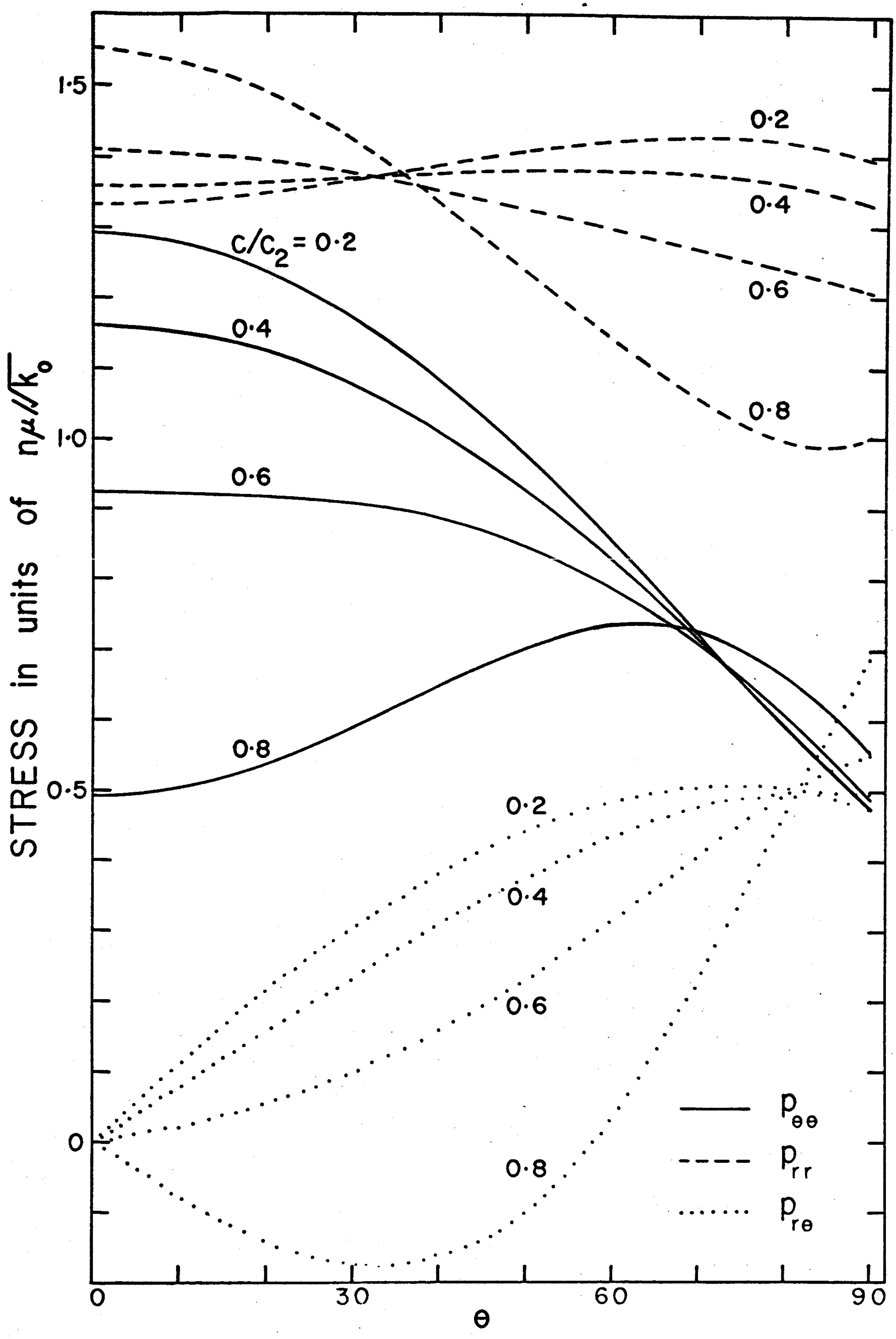

Fig. 3. Stress distribution near the front of a moving Griffith crack of unvarying shape for a medium with a Poisson's ratio of 0.25.

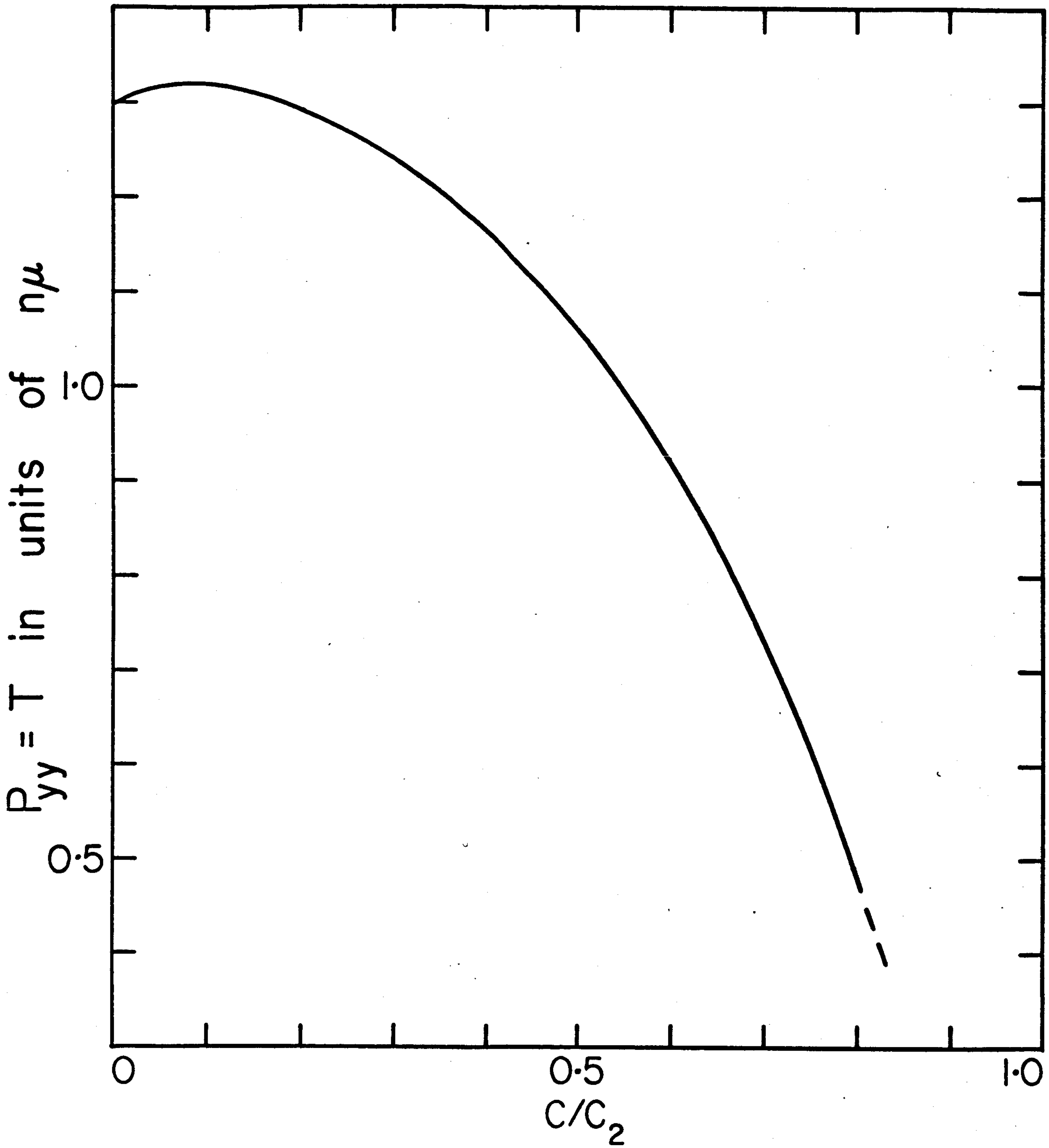

Fig. 4. Applied stress necessary to move a Griffith crack of unvarying shape at different velocities. The medium has a Poisson's ratio of 0.25.